



GRADE 12 FUNCTIONS

WHAT IS A FUNCTION?

A **function** is a relationship between two sets of values where **each input has exactly one output**.

Think of a function as a **machine**: you put an input x into the machine, and it produces exactly one output $f(x)$.

4 DIFFERENT TYPES OF FUNCTIONS

- 1) Linear
- 2) Parabola/Quadratic
- 3) Hyperbola
- 4) Exponential

WHAT IS A LINEAR FUNCTION

Linear Functions

Key idea:

A linear function produces a straight-line graph.

$$f(x) = mx + c$$

Where:

m = gradient

c = y -intercept

• • •

A **linear function** is a function whose graph is a **straight line**.

The general form is:

$$\boxed{y = mx + c}$$

Example:

$$f(x) = 2x + 3$$

Therefore:

$$m = 2, c = 3$$

Remember:

m controls the **steepness and direction** of the line, while c determines where the line crosses the y -axis.

GRADIENT

The **gradient** describes the steepness and direction of a straight line.

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

$m > 0$: line increases

$m < 0$: line decreases

$m = 0$: horizontal line

Undefined: vertical line

INTERCEPTS

y-intercept: where the graph crosses the y -axis.

Set:

$$\boxed{x = 0}$$

For $y = mx + c$:

$$\boxed{y\text{-intercept} = (0, c)}$$

x-intercept: where the graph crosses the x -axis.

Set:

$$\boxed{y = 0}$$

FINDING THE EQUATION

The basic form is:

$$\boxed{y = mx + c}$$

Given gradient and y-intercept:

Substitute directly.

Given two points:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

then substitute into:

$$y = mx + c$$

Given a point and gradient:

$$y - y_1 = m(x - x_1)$$

PARALLEL AND PERPENDICULAR LINES

Parallel:

$$\boxed{m_1 = m_2}$$

Same gradient.

Perpendicular:

$$\boxed{m_1 m_2 = -1}$$

Therefore:

$$m_2 = -\frac{1}{m_1}$$

LINEAR APPLICATIONS

Linear functions model situations with a **constant rate of change**.

$$\boxed{y = mx + c}$$

m = rate of change

c = initial value

Example:

Taxi cost:

$$C(x) = 5x + 20$$

R5 per km + R20 starting fee.

PARABOLA / QUADRATIC

$$F(X)=X^2$$

SHIFTING THE PARABOLA

- $F(x)=a(x-p)^2+q$

A represents the shape of the graph

- P represents the horizontal shift
- Q represents the vertical shift

We can also represent this as :

- $F(x)=ax^2+bx+c$
- A is the shape
- C is the y-intercept

HOW TO DRAW A PARABOLA?

YOU WILL NEED THE FOLLOWING

- 1) X intercept(let $y=$)
- 2) Y intercept(let $x=0$)
- 3) Turning point ($F(-b/2a)$)
- 4) You also need to know how to complete a square!!

PRACTICE AND EXAMPLES

$$f(x)=x^2-6x+5$$

- 1.1 Determine the y -intercept.
- 1.2 Determine the x -intercepts.
- 1.3 Determine the turning point.
- 1.4 Determine the axis of symmetry.
- 1.5 State the domain.
- 1.6 State the range.
- 1.7 State whether the function has a maximum or minimum value.

• • •

$$f(x) = 2x^2 - 8x + 6$$

2.1 Write $f(x)$ in factorised form.

2.2 Write $f(x)$ in turning-point form

...

$$f(x) = -x^2 + 4x + 5$$

- 3.1 Determine the x -intercepts.
- 3.2 Determine the y -intercept.
- 3.3 Determine the turning point.
- 3.4 Determine the axis of symmetry.
- 3.5 State the range.
- 3.6 Sketch the graph of f .

HYPERBOLIC FUNCTION

HYPERBOLIC FUNCTION

A **hyperbolic function** has a graph called a **hyperbola**.

The basic form is:

$$\boxed{y = \frac{a}{x}}$$

where:

$$a \neq 0$$

The Grade 12 transformed form is:

$$y = \frac{a}{x+p} + q$$

BASIC HYPERBOLA

$$y = \frac{a}{x}$$

the graph has **two separate branches**.

If $a > 0$:

Branches are in:

- 1st quadrant
- 3rd quadrant

If $a < 0$:

Branches are in:

- 2nd quadrant
- 4th quadrant

ASYMPTOTES

An **asymptote** is a line that the graph approaches but does not touch.

For:

$$y = \frac{a}{x}$$

the asymptotes are:

$$\boxed{x = 0}$$

and

$$\boxed{y = 0}$$

These are also the **y-axis** and **x-axis**.

TRANSFORMED HYPERBOLA

The Grade 12 form is:

$$y = \frac{a}{x + p} + q$$

The parameters determine the position and shape of the graph:

$a \rightarrow$ shape/orientation

$p \rightarrow$ horizontal shift

$q \rightarrow$ vertical shift

Most important: learn the equation from the asymptotes.

ASYMPTOTES OF THE TRANSFORMED HYPERBOLA

Given:

$$y = \frac{a}{x + p} + q$$

Vertical asymptote:

$$x = -p$$

Horizontal asymptote:

$$y = q$$

Example:

$$y = \frac{2}{x - 3} + 4$$

Here:

$$p = -3, q = 4$$

Therefore:

$$x = 3$$

and

$$y = 4$$

Don't confuse p with the actual asymptote.

DOMAIN AND RANGE

For:

$$y = \frac{a}{x + p} + q$$

the denominator cannot equal zero.

Therefore:

$$x + p \neq 0$$

$$\boxed{x \neq -p}$$

Domain:

$$\boxed{x \in \mathbb{R}, x \neq -p}$$

Range:

$$\boxed{y \in \mathbb{R}, y \neq q}$$

Remember:

Domain $\rightarrow$ restriction on x

Range $\rightarrow$ restriction on y

INTERCEPTS

To find the x -intercept, set:

$$y = 0$$

To find the y -intercept, set:

$$x = 0$$

Example

$$y = \frac{2}{x-1} + 3$$

For the y -intercept:

$$y = \frac{2}{0-1} + 3 = 1$$

Therefore:

$$\boxed{(0, 1)}$$

For the x -intercept:

$$0 = \frac{2}{x-1} + 3$$

Then solve for x .

AXES FOR SYMMETRY

A hyperbola has **two axes of symmetry**.

For:

$$y = \frac{a}{x + p} + q$$

the centre is:

$$\boxed{(-, pq)}$$

The axes of symmetry are:

$$\boxed{y - q = x + p}$$

and

$$\boxed{y - q = -(x + p)}$$

These pass through the **centre of the hyperbola**.

READING A HYPERBOLA FROM A GRAPH

To sketch:

$$y = \frac{a}{x + p} + q$$

Step 1

Identify the asymptotes:

$$x = -p$$

$$y = q$$

Step 2

Draw the asymptotes as **dashed lines**.

Step 3

Find the orientation using the sign of a .

Step 4

Find useful points/intercepts if required.

Step 5

Draw the two branches approaching the asymptote

EXPONENTIAL FUNCTIONS

EXPONENTIAL FUNCTIONS

An **exponential function** is a function where the variable occurs in the exponent.

The basic form is:

$$\boxed{f(x) = b^x}$$

where:

$$b > 0, b \neq 1$$

EXPONENTIAL LAWS

You need the exponent laws from Grade 11.

$$a^m \times a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^0 = 1$$

These laws are essential for simplifying and solving exponential equations.

THE BASIC EXPONENTIAL GROWTH

There are two important cases.

If $b > 1$

The graph represents **growth**.

If $0 < b < 1$

The graph represents **decay**.

KEY FEATURES

$$y=b^x$$

y -intercept

Set $x = 0$:

$$y = b^0 = 1$$

Therefore:

$$\boxed{(0, 1)}$$

x -intercept

There is **no x -intercept**.

Asymptote

$$\boxed{y = 0}$$

Domain

$$\boxed{x \in \mathbb{R}}$$

Range

$$\boxed{y > 0}$$

TRANSFORMATIONS

The general transformed exponential function is:

$$\boxed{y = ab^{x+p} + q}$$

The parameters affect the graph.

a

Controls:

vertical stretch/compression

reflection in the x -axis

p

Controls:

horizontal shift

q

Controls:

vertical shift

INVERSE FUNCTIONS

INVERSE FUNCTIONS

An inverse function reverses the effect of a function.

If:

$$f(x) = y$$

then the inverse takes y back to x :

$$\boxed{f^{-1}(y) = x}$$

Important:

$$\boxed{f^{-1}(x) \neq \frac{1}{f(x)}}$$

FINDING THE INVERSE

To find $f^{-1}(x)$:

1. Write:

$$y = f(x)$$

2. Interchange x and y .

3. Solve for y .

4. Replace y with $f^{-1}(x)$.

DOMAIN AND RANGE

For an inverse:

$$\boxed{\text{Domain of } f = \text{Range of } f^{-1}}$$

and

$$\boxed{\text{Range of } f = \text{Domain of } f^{-1}}$$

So the inverse swaps the domain and range.

GRAPH OF INVERSE

he graph of f^{-1} is a reflection of f in the line

$$\boxed{y = x}$$

If:

$$(a, b)$$

lies on f , then:

$$(b, a)$$

lies on f^{-1} .

So the x - and y -coordinates are swapped.

REMEMBER

$f^{-1}(x)$ reverses $f(x)$

Swap x and y to find the inverse

Domain and range are interchanged

Graphs reflect in $y = x$

A function must be one-to-one

And the four important examples:

Linear $\rightarrow$ Linear

Parabola $\rightarrow$ Square root

Hyperbola $\rightarrow$ Hyperbola

Exponential $\rightarrow$ Logarithmic